International Journal of Advanced Research in Engineering Technology & Science
ISSN: 2349-2819
Email: editor@ijarets.org Volume-3, Issue-12 December- 2016 www.ijarets.org

EFFECT OF HALL CURRENT ON UNSTEADY TRANSVERSE MAGNETIC FIELD ON THE FLOW OF A CONDUCTING DUSTY GAS DUE TO AN IMPULSIVELY STARTED FLAT PLATE

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ABSTRACT: In this paper, we study on unsteady laminar rotating flow of a viscous incompressible and electrically conducting gas suspended with a uniform distribution of non conducting dust particles in presence of a transverse magnetic field fixed relative to the plate is considered. We find that the velocity distribution of the conducting dusty gas, and non conducting dust particle increases with the increase of Hartmann number (M).But Hall parameter (m) increases the as velocity of the fluid decreases. It is also found from the table the skin friction increase with the increase of (m at l =.4 and M =.5), skin friction decreases with the increase of M at l = .4.

INTRODUCTION

The problem of hydro magnetic flow in a rotating channel in the presence of an applied uniform magnetic field finds its applications in induction flow meters. MHD generators or accelerators constructions of the turbines and other centrifugal machines. Seth and Jana (1980) have studied the unsteady hydro magnetic flow in a rotating channel neglecting Hall currents. However, in the presence of strong magnetic field the Hall currents play a significant role in determining the flow features.

Russow (1958) has studied the flow of a viscous incompressible and electrically conducting fluid over a flat plate in the presence of a uniform transverse magnetic field, which is fixed relative to the plate or to the fluid.

Soundalagekar (1965) , Pop(1968) and Pande (1970) have solved the hydro magnetic flow due to accelerated motion of an infinite flat plate in the presence of magnetic field fixed relative to the plate. A similar problem, when the magnetic field is fixed relative to the fluid, has been solved by Gupta (1960), Baral (1968) studied the flow of a conducting dusty gas occupying a semi infinite space in the presence of transverse magnetic field fixed relative to the fluid. It has been pointed out by Rao and Murthy (1972) that there was an error in the paper of Baral (1968). Recently, Mitra (1980) studied the effect of a magnetic field on the flow of an electrically conducting dusty gas over an oscillating flat plate.

P. Mitra has studied effect of transverse magnetic field on the flow of a conducting dusty gas due to an impulsively started flat plate. In the present paper we have studied the effect of Hall parameter on the dusty gas.

LIST OF SYMBOLS

a	radius of a spherical dust particle;
B_0	The uniform transverse magnetic field;
k	stokes resistance coefficient;
l	mass of a dust particle;
M	the magnetic field parameter (Hartmann number);
m'	mass of a dust particle;
N_0	the number density of dust particle;
t	time;
U	constant velocity of the plate;

u, v velocity components of the conducting dusty gas and dust particle parallel to the plate respectively;
y rectangular coordinate normal to the plate;
ν Kinematic coefficient of viscosity of the conducting dusty gas;
μ viscosity of conducting dusty gas;
σ electrical conductivity of the conducting dusty gas;
τ the relaxation time of the dust particle;
ρ the density of the conducting dusty gas;
m Hall parameter.

FORMULATION OF THE PROBLEM

Consider x – axis along the plate and y – axis normal to it. The time dependent equation of laminar moving of the conducting dusty gas and non conducting dust particles, after introducing the electromagnetic force in the equation according to Baral (1968) used :-

$$\frac{\partial u}{\partial t} + (u.\nabla)u = -\frac{1}{\rho}\nabla P - \nu\nabla^2 u + \frac{KN_0}{\rho}(v-u) + \frac{1}{\rho}(JX\,H) \quad \dots (1)$$

$$m'\left[\frac{\partial v}{\partial t} + (v.\nabla)v\right] = K(u-v) \quad \dots (2)$$

div u = 0 …(3)

$$\frac{\partial N_0}{\partial t} + div(N.v) = 0 \quad \dots (4)$$

$$\nabla XE = 0 \;\; , \nabla XJ = 0$$

$$J = \sigma\left[E + \mu_e\, uXH - \frac{\mu}{\eta_e\, e} JXH\right] \quad \dots (5)$$

J and H are given by Maxwell's equation. Since the plate is semi infinite in the x and y directions, the velocity field is assumed to be function of y and t only, it is assumed that the induced magnetic field is negligible so that H = (0, H_0 ,0), this assumption is justified since the magnetic Reynold number is very small.

For the gas embedding small solid particle in equation of conservation of electric charge J = 0 gives, J_y = constant, when J = (J_x, J_y, J_z)

This constant is zero since J_y = 0 at the plate, which is electrically non conducting. Thus J_y = 0 everywhere in the flow.

We consider here the "short circuit" case so E = 0, under these assumptions, the equation of motion of the conducting dusty gas and the dust particle along the axis of X is now governed by –

$$\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{KN_0}{\rho}(v-u) - \frac{\sigma\mu_e^2 H_0^2}{1+m^2}u^* \quad \dots (6)$$

$$\tau\frac{\partial v}{\partial t} = (u-v) \quad \dots (7)$$

Where $\tau = \frac{m'}{K}$

The equation (6) and (7) can be put in non dimensional forms with the help of the following substitutions –

$$t = \frac{t^*}{\tau}, y = \frac{y^*}{(\nu\tau)^{\frac{1}{2}}}, u = \frac{U^*}{U}, v = \frac{v^*}{U}, \sigma\mu_e^2 H_0^2\tau = M^2$$

m (be the Hall parameter)

The equation (6) and (7) reduce to (omitting the asterisks)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + l(v-u) - \frac{M^2}{1+m^2}u \quad \dots (8)$$

And $\frac{\partial v}{\partial t} = (u-v)$ … (9)

Where $l = \frac{m' N_0}{\rho}$

Now when the magnetic field is relative to the plate, then equation (1) will be modified. At t = 0, the conducting dusty gas the plate and the magnetic field are moving with velocity u = 1. Because the magnetic field is moving and the conducting dusty gas is initially at rest. The relative motion must be accounted for (since the origin of co-ordinate is fixed in space). Hence, by transformation of co-ordinates, the equation (8) becomes –

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + l(v-u) - \frac{M^2}{(1+m^2)}(u-1) \qquad \dots (10)$$

Equation (10) and (9) are to solve under the initial and boundary conditions:-

$$t = 0,\ u = v = 0 \quad \text{for } y > 0 \qquad \dots (11)$$

$$t > 0, u = 1 \quad at\ y = 0$$

$$u \to \infty\ as\ y \to \infty \qquad \dots (12)$$

Let, $\bar{u} = \int_0^\infty u e^{-st}\, dt, \quad \bar{v} = \int_0^\infty v e^{-st}\, dt \quad (\text{Re } s > 0)$

Be respectively the Laplace transform of u and v multiplying (10) and (9) by e^{-st} and integrating we get using equation (11)

$$\frac{\partial^2 \bar{u}}{\partial y^2} - P^2 \bar{u} = -\left[\frac{M^2}{1+m^2}\right]\frac{1}{s} \qquad \dots (13)$$

And $\bar{v} = \frac{\bar{u}}{1+s} \approx \bar{u} \quad s \ll 1 \qquad \dots (14)$

Where $P^2 = s + \left[\frac{M^2}{1+m^2}\right] + \frac{l.s}{1+s} \qquad \dots (15)$

The boundary conditions (12) transform to:-

$$\bar{u} = \frac{1}{s} \quad \text{at } y = 0, \quad \bar{u} \to finite\ as\ y \to \infty \qquad \dots (16)$$

The solution of (13) subject to the conditions (16)

$$\bar{u} = \left[\frac{1}{s} - \frac{M^2}{(1+m^2)P^2 s}\right] e^{-Py} + \frac{1}{P^2}\frac{M^2}{(1+m^2)\, s} \qquad \dots (17)$$

Since the inversion of equation (17) presents some difficulty so we restrict ourselves to large value of t, now, when" t , is large , then " s , is very small (Gustav Doetsch 1943).

Therefore in this case

$$P = \left[(1+l)\, s + \frac{M^2}{1+m^2}\right]^{\frac{1}{2}} \qquad \dots (18)$$

Then from equation (17) we get (for large t)

$$\bar{u} = \frac{1}{s} - \frac{1}{s + \left(\frac{M^2}{1+m^2}\right)\left(\frac{1}{1+l}\right)} + (1+l)\left[\frac{1}{(1+l)\, s + \frac{M^2}{1+m^2}}\, e^{-\left[(1+l).s\,\frac{M^2}{1+m^2}\right]^{\frac{1}{2}} y}\right] \qquad \dots (19)$$

∴ By inversion theorem we get

$$u = 1 - e^{-\frac{M^2 t}{(1+m^2)}\frac{1}{(1+l)}} + (1+l)\,\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{st}\,\frac{1}{(1+l).s + \frac{M^2}{1+m^2}}\, e^{-Py} ds \qquad \dots (20)$$

Let $D = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{st}\,\frac{1}{(1+l).s + \frac{M^2}{1+m^2}}\, e^{-Py} ds$

Now, $(1+l)s + \frac{M^2}{1+m^2} = P^2$

$$\therefore D = e^{-\frac{M^2 t}{(1+m^2)(1+l)}} . \frac{1}{\pi i}\int_{Br_3} \frac{e^{\frac{P^2 t}{(1+l)}}}{P}\, dP \qquad \dots (21)$$

Where Br_3 is the Bromwich path defined by Mclachlah (1953).

Again we put $z = \frac{P^2}{1+l}$

Then equation (21) reduce to

$$D = \frac{e}{(1+l)}^{-\frac{M^2 t}{(1+m^2)(1+l)}} \cdot \frac{1}{2\pi i} \int_{Br_1} \frac{e^{zt-(1+l)^{\frac{1}{2}} y z^{\frac{1}{2}}}}{z} \, dz \qquad \ldots (22)$$

Integrating along Br1 we have (Mclachlah 1953)

$$\frac{1}{2\pi i} \int_{Br_1} \frac{e^{zt-(1+l)^{\frac{1}{2}} y z^{\frac{1}{2}}}}{z} \, dz = erfc\left[\frac{y(1+l)^{\frac{1}{2}}}{z.\sqrt{t}}\right] \qquad \ldots (23)$$

Thus finally (20) reduce to

$$u = 1 - \frac{e}{(1+l)}^{-\frac{M^2 t}{(1+m^2)(1+l)}} + \frac{e}{(1+l)}^{-\frac{M^2 t}{(1+m^2)(1+l)}} . erfc\left[\frac{y(1+l)^{\frac{1}{2}}}{2.\sqrt{t}}\right] \qquad \ldots (24)$$

Similarly, taking inverse transform, for large value of t,

We get from equation (14)

$$v = u - e^{-\frac{M^2 t}{(1+m^2)(1+l)}} \left[\frac{M^2}{(1+m^2)(1+l)} erfc\left[\frac{y(1+l)^{\frac{1}{2}}}{2.\sqrt{t}}\right] + \frac{y}{2}\left(\frac{1+l}{\pi+3}\right)^{\frac{1}{2}} . e^{\frac{-y^2(1+l)}{4t}}\right] \to u \qquad \ldots (25)$$

Equation (24) and (25) represent respectively the velocities of conducting dusty gas and non conducting dust particle for large t in the case when the magnetic field is fixed relative to the plate. Putting M = 0 in equations (24) and (25) we get,

$$u = erfc\left[\frac{y(1+l)^{\frac{1}{2}}}{2.\sqrt{t}}\right] \qquad \ldots (26)$$

And

$$v = erfc\left[\frac{y(1+l)^{\frac{1}{2}}}{2.\sqrt{t}}\right] - \frac{y}{2}\left(\frac{1+l}{\pi+3}\right)^{\frac{1}{2}} . e^{\frac{-y^2(1+l)}{4t}} \qquad \ldots (27)$$

Results (24) and (25) agree with the results of Liu (1967). Again putting $M = 0 \; and \; l = 0$ in equation (24) we get

$$u = erfc\left(\frac{y}{2\sqrt{t}}\right) \qquad \ldots (28)$$

Which is well known solution for Stokes" first problem in classical hydrodynamics.

CALUCLATION OF THE SKIN FRICTION

For finding the shearing stress at the plate from equation (24), one needs the following result for the complimentary error function (Milton and Stegun, 1964).

$$\frac{d^{h+1}}{dz^{h+1}} [erfc(z)] = (-1)^h \frac{2}{\sqrt{\pi}} H_h(z) e^{-z^2} \qquad \ldots \quad (29)$$

(h = 0, 1, 2…)

Then using (29), when t large, the skin friction at the plate is obtain as

$$D_0 = \left(-\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\sqrt{(1+l)}}{\pi t} - e^{-\frac{M^2 t}{(1+m^2)(1+l)}} \qquad \dots (30)$$

RESULT AND DISCUSSION

In order to get a physical understanding of the problem which are generally valid for large values of t, numerical calculations are carried out for the velocities of the conducting dusty gas,

Table (I) Velocity of the conducting dusty gas for t = 2, l = .1 from equation (24)

Y	M = .5 m = 2	M = 1 m = 2	M = 1 m = 1.5	M = .5 m = 1.5	M = 1 m = 1	M = .5 m = 1
0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.1	0.96614	0.97422	0.97881	0.96776	0.98506	0.97046
0.2	0.93228	0.94845	0.95762	0.93552	0.97012	0.94091
0.3	0.89833	0.92259	0.92999	0.89449	0.95065	0.90242
0.4	0.84749	0.88389	0.90454	0.85477	0.93270	0.88911
0.5	0.79665	0.84519	0.87272	0.80636	0.91027	0.82257
0.6	0.77632	0.82971	0.85999	0.78699	0.90130	0.80483
0.7	0.72887	0.79359	0.83029	0.75795	0.88036	0.76343
0.8	0.70274	0.77370	0.81394	0.71693	0.86884	0.74064
0.9	0.67302	0.74855	0.79533	0.68863	0.85572	0.71470
1.0	0.60884	0.70221	0.75516	0.62751	0.82740	0.65870

Non conducting dust particle and the skin friction at the plate from the equation (24), (25) and (30) respectively and the result shown by tables. From table (i), we find the velocity distribution of the conducting dusty gas for different values of Hartmann number (M) and Hall parameter (m) at different point of flow fields, it is noted that for fixed l = .1 and t = 2 the velocity of conducting dusty gas increase with the increase of M. But the Hall parameter increase, the velocity of conducting dusty gas decrease.

From table (ii) velocity distribution the non conducting dust particle for different values of M and m for l = .1, t = 2 at different points of the flow fields, velocity of the non conducting dust particle increase with the increase of M at fixed value of m. But velocity of non conducting dust particle decreases with increase of m at fixed value of M.

From table (iii) the skin frictions at the plate are shown for different of M, m and l at fixed value of t = 2, first results that for fixed values l = .4, M = .5 the skin friction increase of m. And the second result that skin friction decrease with increase of Hartmann number M at l = .4. Third result that for fixed value of Hartmann number (M = .5), the skin friction increase with the increase of l.

Table (ii) Velocity of the dust particle for t = 2, l = .1 from equation (25)

Y	M = .5 m = 2	M = 1 m = 2	M = 1 m = 1.5	M = .5 m = 1.5	M = 1 m = 1	M = .5 m = 1
0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.1	0.94530	0.954886	0.96080	0.95209	0.96976	0.98580
0.2	0.89077	0.90985	0.92171	0.89501	0.93958	0.97717
0.3	0.82578	0.85576	0.87458	0.83154	0.90296	0.95475
0.4	0.76497	0.80523	0.83052	0.77263	0.86876	0.93890
0.5	0.69724	0.74598	0.77868	0.70391	0.82831	0.92039
0.6	0.65581	0.71476	0.75177	0.66705	0.80776	0.91053
0.7	0.59010	0.67505	0.70369	0.60336	0.77020	0.89337
0.8	0.50462	0.61235	0.67330	0.56235	0.74677	0.88237
0.9	0.45523	0.58737	0.64213	0.51876	0.72150	0.87063
1.0	0.42265	0.51987	0.58129	0.44116	0.67465	0.84964

Table (iii) Skin Friction at the plate for t = 2, from equation (30).

m	l = .4 M = .5	l = .4 M = 1	l = .8 M = 1.5	l = .8 M = 1	l = .8 M = .5
0	0.33027	0.11312	0.04393	0.17618	0.40539
1	0.39488	0.23108	0.15334	0.30707	0.41679
2	0.43949	0.35472	0.32462	0.42855	0.50628
3	0.45547	0.41705	0.41681	0.47892	0.52054

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